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Brownian Motion: A Classroom Demonstration and Student Experiment

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Lavenda (*1*) has described how Einstein and Perrin employed Brownian motion to confirm an atomic theory of matter. Teachers may also use Brownian motion for instruction in the classroom and laboratory. This paper will show how video recordings of the Brownian motion of tiny particles may be made, describe a classroom demonstration, cite a reported experiment designed to show the random nature of Brownian motion, and suggest a student experiment to discover the distance that a tiny particle travels as a function of time. The depth of the statistical analysis of the data can easily be adjusted to the level of the class.

Recording Brownian Motion

A video cassette recording of the projection onto the x–y plane of the Brownian motion of a tiny particle was made in the following way. One drop of a 5% suspension of latex spheres 1.099 μm in diameter was mixed with 3 L of water. A drop of this dilute suspension was placed on a hemacytometer that had line markings 0.05 mm apart. The suspension was covered with a glass slide, and the hemacytometer was mounted on a compound microscope that had a trinocular head.

A Petri dish filled with an aqueous solution of sodium carbonate was placed over the light source of the microscope to absorb infrared radiation in order to inhibit the formation of convection currents that would cause bulk motion within the suspension. The stage of the microscope was carefully leveled.

Because the concentration of latex spheres in the suspension was very low, it was easy to isolate and observe only one sphere in the microscope's field of view. However, with only one, or perhaps two, spheres in view, it is impossible to detect if there is bulk motion in the suspension.

A Panasonic color camera (model WV-3320) was mounted on the microscope, which recorded the movement of a latex sphere on a Sony Beta II/III video cassette recorder (model SL-5400). The picture was also simultaneously displayed on the screen of a 19-in. NEC Autocolor monitor (model CM-1951A).

Because the latex particle is moving along the verticle axis as well as in the x–y plane, its image must continuously be kept in focus by gently turning the microscope's focus knob. Even so, the operator must be very careful to keep the stage of the microscope level and fixed in the x–y plane while recording the Brownian motion of a particle.

The markings 0.05 mm apart on the hemacytometer were distorted on the monitor's screen. A distance of 0.05 mm appeared as 178 mm on the screen's vertical axis and 191 mm on its horizontal axis. This represents a distortion of about 7% and an average magnification of about 3700×.

The important pedagogical question is "What use can be made of video recordings of Brownian motion?"

Classroom Demonstration

The equipment described above may be employed to show the actual Brownian motion of tiny particles to an entire class so that all students are able to view the motion simultaneously. This would not be possible if each student had to observe the motion individually through the eyepiece of a microscope. Also, the teacher may easily and rapidly change the objective lens of the microscope in order to increase or decrease the field of view and the magnification of the particles displayed on the monitor. In less than a minute, a teacher could remove the hemacytometer in order to change the suspension on it so that the Brownian motion of a suspension of carbon black or dye particles in water might be displayed on the monitor.

Students are able to observe several important characteristics of Brownian motion in this demonstration. If many particles are displayed at once, the ceaseless motion of the particles is random. The motion of one particle is not affected by the motion of another particle even if the two particles are within one particle diameter of each other. Particular particles will eventually move in and out of focus indicating that Brownian motion occurs in three dimensions. Moving the suspension up and down along the vertical axis by use of the focus knob will allow students to observe that the concentration of particles diminishes with increasing height in the suspension. If the particles undergoing Brownian motion are changed, it is apparent that the rapidity of the motion becomes greater as the particles become smaller.

If, during a demonstration, the particles appear in concert to move slowly toward one side of the monitor's screen, then the particles are undergoing bulk motion in addition to Brownian motion. For this reason, a teacher may substitute videotape recordings of Brownian motion made under carefully controlled conditions in place of observing Brownian motion directly. The videotape recording also preserves this demonstration so that it can be easily and confidently used in future classes.

The videotape recordings may be temporarily stopped (pause) and started again (play) at the discretion of the teacher. This enables tracks of these tiny particles to be drawn. This is accomplished by taping a transparent acetate sheet over the monitor's screen and marking the point of origin of the particle's motion on this sheet while the videotape is in the pause mode. The videotape is then begun and allowed to run for a designated period of time, say 10 seconds. At that time the videotape is stopped and the new position of the particle is marked on the acetate sheet. One could repeat this process at equal intervals of time until the particle's image leaves the edge of the monitor's screen.

If the position of the latex particle is marked at 10-s intervals for 2 min, the acetate sheet will have 13 points that may be connected consecutively by 12 straight lines to give a crude representation of the track of the particle. Tracks of this type are often shown in textbooks and clearly show the fluctuating nature of Brownian motion because no two

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Table 1. **Values for the Root-Mean-Square Displacements Calculated from the Polynomials Given in Table 2 for Various Values of the Square Root of Time Compared to Experimental Data**

$t^{1/2}$ ($s^{1/2}$)	Root-mean-square Displacement (μm)					
	Expt.	linear	second	third	fourth	fifth
0.00	0.00	0.00	0.43	−0.07	−0.01	0.00
3.16	3.60	4.36	3.19	3.97	3.68	3.58
4.47	5.35	6.16	4.79	5.32	5.34	5.44
5.48	6.71	7.56	6.21	6.42	6.58	6.66
6.32	7.85	8.72	7.52	7.44	7.62	7.62
7.07	8.22	9.75	8.78	8.49	8.61	8.55
7.75	9.65	10.69	9.99	9.58	9.60	9.53
8.37	10.46	11.54	11.17	10.71	10.64	10.59
8.94	11.84	12.33	12.30	11.89	11.75	11.75
9.49	13.30	13.09	13.44	13.18	13.01	13.06
10.00	14.23	13.79	14.54	14.50	14.38	14.46
10.49	15.94	14.47	15.64	15.91	15.91	15.96
10.95	17.55	15.10	16.70	17.37	17.59	17.50

tracks are alike. A teacher may make as many of these tracks as he or she wishes by cleaning the acetate sheet with a cloth dampened with alcohol and repeating the tracking demonstration with another particle's motion that has been recorded on the videotape.

Simulation of Brownian Motion

A student experiment to simulate Brownian motion has been reported (2). This experiment employs 12 dice and a hexagonal grid in order to simulate the track of a randomly moving particle. The dice are rolled in order to determine the movement of a point. Straight lines are drawn on the grid to connect the positions of the moving point after each roll of the dice. The fluctuations that occur in the direction and length of the vectors that describe the point's motion are clearly shown on the grid. When the tracks of the points on the hexagonal grid are compared to the tracks of Brownian particles obtained from the videotapes, the similarities are unmistakable even though no two tracks are exactly alike.

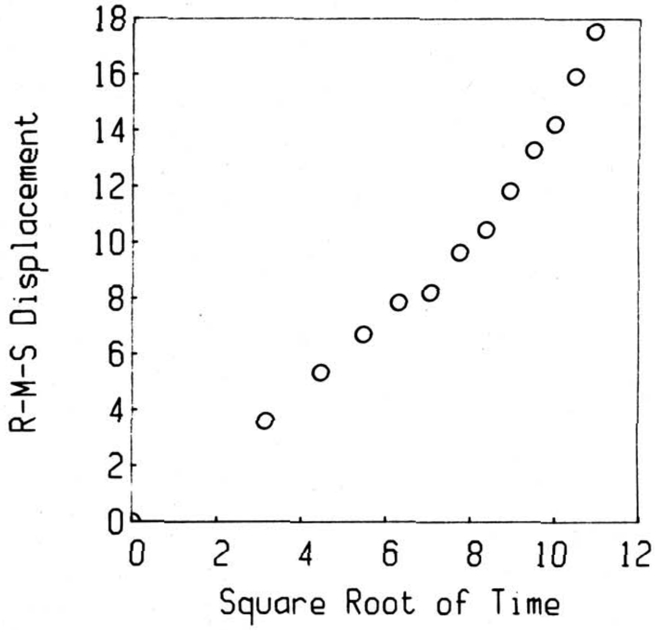
Student Experiment

The tracks of many individual particles can be drawn by replaying the video recordings of these particles. Microcomputers can readily perform statistical calculations on measurements made from these tracks. Hence, a student experiment may be performed to discover the relationship between the root-mean-square displacement of these particles and time.

In one experiment the tracks of 21 particles were recorded at 24 °C by marking the position of each particle every 10 s for a duration of 120 s. The radial distances were measured from the origin to each of the 13 marked points on every track. The average root-mean-square displacements at each 10-second interval were calculated (first two columns of Table 1) and plotted as a function of the square root of time (figure) by use of a Lotus 1–2–3 software program.

Do the data plotted in the figure describe a mathematical function, and if so, what is this function? Some students may feel that these data points can be adequately represented by a linear function, but others may be skeptical because of the obvious upward curve of these points at higher values of time. These different opinions must be taken seriously in a science class. They can provide an opportunity for teaching students about the uncertainties in scientific experiments and the statistical treatment of data.

Students can see that the data points on the graph clearly do not lie on a straight line, but they also know that there is experimental error in every measurement described by these



The average root-mean-square displacement (microns) for 21 latex particles (dia. 1.099 μm) plotted as a function of the square root of time ($s^{1/2}$) at 24 °C.

Table 2. Coefficients for Various Polynomials[a]

Degree	A_0	A_1	A_2	A_3	A_4	A_5
linear[b]	+0.00	+1.38				
second	+0.43	+0.63	+0.079			
third	−0.074	+1.67	−0.17	+0.015		
fourth	−0.0089	+0.82	+0.20	−0.037	+0.0022	
fifth	+0.00028	−0.26	+0.89	−0.19	+0.017	−0.00049

[a] Polynomials have the form $\langle rms \rangle_{xy} = A_0 + A_1x + \cdots + A_5x^5$ where $x = t^{1/2}$.
[b] This linear fit was constrained to pass through the origin.

data points except for the one at the origin. They know that there is approximately a 7% distortion in the image displayed on the monitor, that there is an obvious error of parallax in marking the position of the tiny particle on the acetate sheet because this sheet lay a few millimeters above the actual picture screen of the monitor, and that measuring the distance between points marked on the acetate sheet is in error by at least, if not more than, the radius of the particle's image on the monitor. After some class discussion and thought about the errors in measurements made in this experiment, it can be estimated that these errors could easily be 10–15%. Consequently, students should realize that every point on the graph represents an honest measurement, but none are the correct representation of natural behavior except for the point at the origin.

Students may select polynomial functions to describe the natural behavior and proceed by calculating the coefficients for various polynomials. For the linear polynomial, the linear regression formula that is constrained to make the straight line pass through the origin was used because the correct function is known to pass through the origin. For polynomials of higher degrees, a linear least-squares polynomial curve-fitting program was used. The calculated coefficients for five polynomials are shown in Table 2. Values for the root-mean-square displacements calculated by use of each polynomial are shown in the last five columns of Table 1.

If the data in Table 1 are examined carefully, the following observations and conclusions could be made. The higher polynomials do generate an average root-mean-square displacement that is closer to the experimental value. For the third and higher degree polynomials, the values generated differ by about 3% or less from the experimental values. However, because we suspect that our data is in error by 10–15%, we conclude that these polynomials are not describing a law of nature but are simply doing a good job of describing measurements that are known to have errors. The quadratic function generates values that can be quickly estimated to differ, on an average, by about 5% from the experimental values. The quadratic equation also has an intercept that does not pass through the origin. The linear function was forced to pass through the origin, but the values generated by use of this function appear to differ from the experimental values by an average of approximately 10%, which is nearer to our expectation for the function that describes a law of nature. Consequently, in the absence of additional data and the existence of an understood risk, we may select the linear function as that one likely to describe a law of nature.

Students may now be introduced to the theoretical explanation for Brownian motion that was derived by Einstein in 1905. An elementary discussion of this work was reported in 1908 and is recorded in English (*3*). The motion of Brownian particles in the x–y plane is described by

$$\langle \mathrm{rms} \rangle_{xy} = 2D^{1/2}t^{1/2}$$

where $\langle \mathrm{rms} \rangle_{xy}$ is the root-mean-square displacement of the particle from its origin at time, t. D is the diffusion coefficient, which depends on the temperature, radius of the diffusing particle, and the viscosity of the liquid in which the particle is suspended. Students may employ this equation and the slope of the linear function to calculate a value of 0.48 $\mu m^2/s$ for the diffusion coefficient of the latex spheres.

How can the students know that this value for the diffusion coefficient is a realistic one? The Stokes–Einstein law gives the relation between these quantities as

$$D = \frac{kT}{6\pi\eta r}$$

where k is the Boltzmann constant, T is the absolute temperature, η the viscosity of the medium, and r is the radius of the particle. When this equation is used to calculate the diffusion coefficient for latex spheres of 1.009 μm diameter in water at 24 °C, a value of 0.43 μm^2 is obtained. The value obtained from the slope of the experimental line is 12% higher than this calculated value.

If an experimental value for the diffusion coefficient at 24 °C matched that calculated from the Stokes–Einstein law, the slope of the experimental line would have to be 1.32 $\mu m/s^{1/2}$ instead of the value of 1.38 $\mu m/s^{1/2}$ that was obtained from the linear function. Comparing these two slopes by use of the "Student t-test" (*4, 5*) provides a way to instruct students in the statistics of a straight line and a criterion to pass judgment on the selection of the linear function to describe a law of nature.

The estimated standard deviation for the slope of the straight line passing through the origin (1.38 $\mu m/s^{1/2}$) was calculated to be 0.041 $\mu m/s^{1/2}$. When the difference in the two slopes was divided by this estimated standard deviation, a value for t of 1.47 was obtained, which is smaller than the t value of 1.78 at the 90% confidence level for 12 degrees of freedom. We can then conclude that more than 10% of the samples selected from the same population would have exhibited slopes that differed more than these two, and the null hypothesis is accepted. Hence, students may use video recordings of the tracks of latex particles to show that the motion of these particles is described by Einstein's equations for diffusion.

Students may obtain an indication of the probable error made in measuring the diffusion coefficient by calculating the propagation of the probable errors that were made in their measurements (*6*). This would produce a statistically calculated measure on either side of the mean in which half the determinations of the diffusion coefficient would be expected to lie. To do this, the video recording of the same latex particle may be rerun several times, and its track may be plotted each time. These independently plotted tracks, which ideally should be identical, may be analyzed to determine the average standard deviations in measuring both time and mean square displacements. These standard deviations were found to be 0.56 s for time measurements and 25 μm^2 for mean-square displacements.

From an analysis of the movement of the 21 particles for 2 min, the average of the mean-square distances was found to be 115 μm^2 and the average time was obviously 60 s. These values were used to calculate a probable error of 0.071 $\mu m^2/s$, which is 15% of the experimentally determined value for the diffusion coefficient (0.48 $\mu m^2/s$).

Teachers may wish to direct students to the work of Jean Baptiste Perrin (*7*), who determined Avogadro's number in 1908 by measuring the distribution of tiny particles in a gravitational field. Perrin reported a value of 7.06×10^{23}, which is in error by 17%. In 1926, Perrin was awarded the Nobel Prize in physics for this work.

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